



# **Fine contribution to mechanical stability and constitutive modelling strategy**

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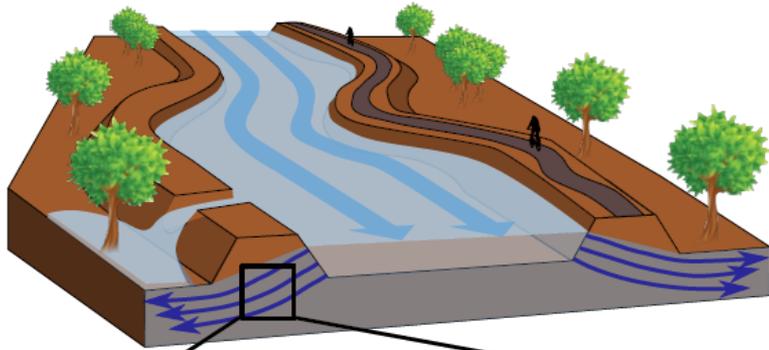
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# > Suffusion relationship with fine grains



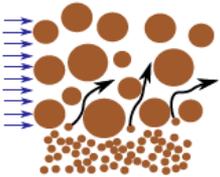
Suffusion = selective erosion of the finest particles of a soil

Internal stability  $\neq$  mechanical stability

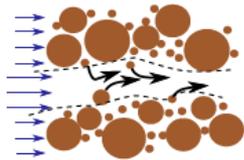
4 types of internal erosion



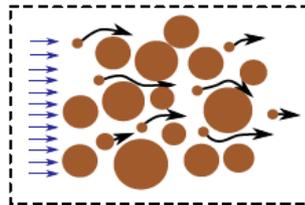
Backward erosion



Contact erosion



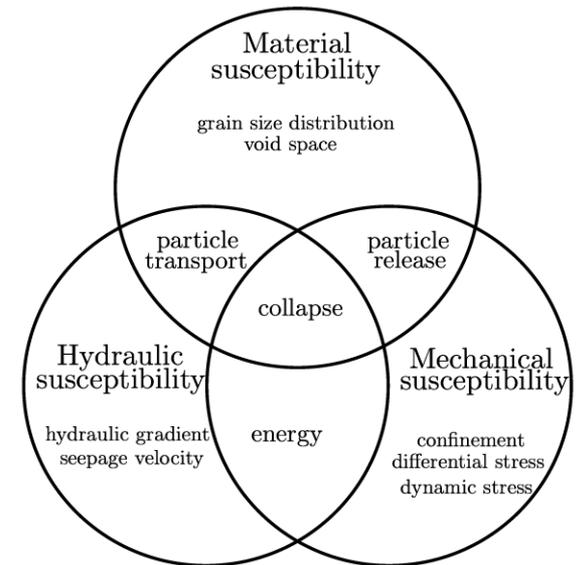
Piping erosion



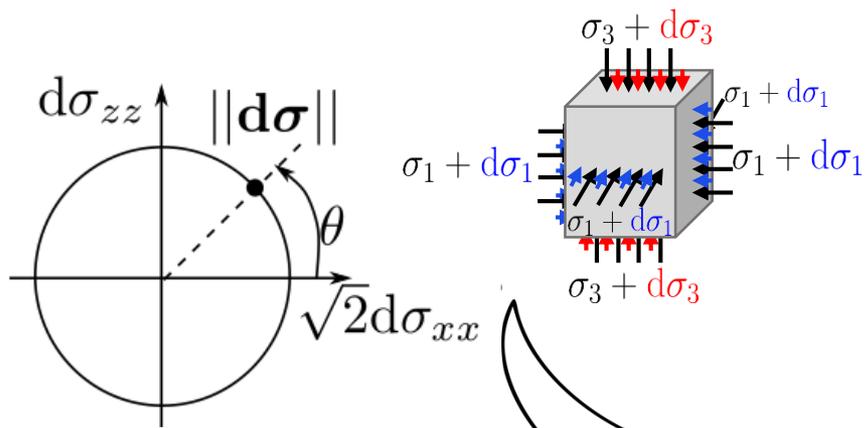
Suffusion

What are the consequences of **suffusion** in terms of **mechanical properties**?

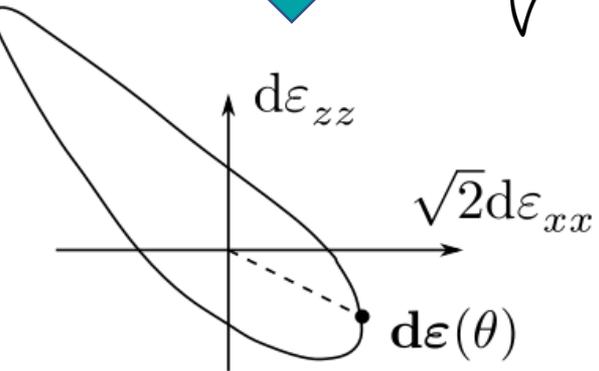
What are the consequences of **fine particles** in terms of **mechanical stability**?



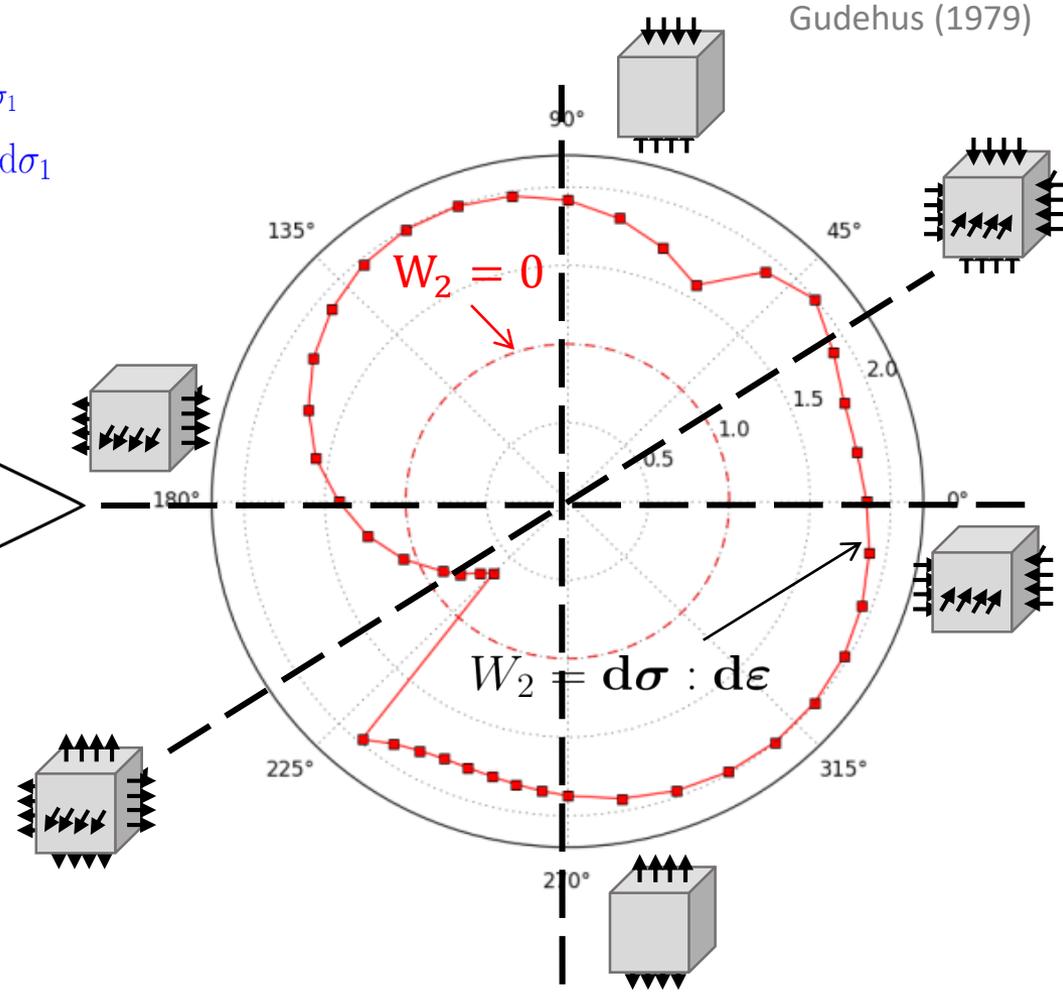
# ➤ Mechanical stability: the second-order work criterion



Incremental stress probes (axisymmetric plane)



Incremental strain responses



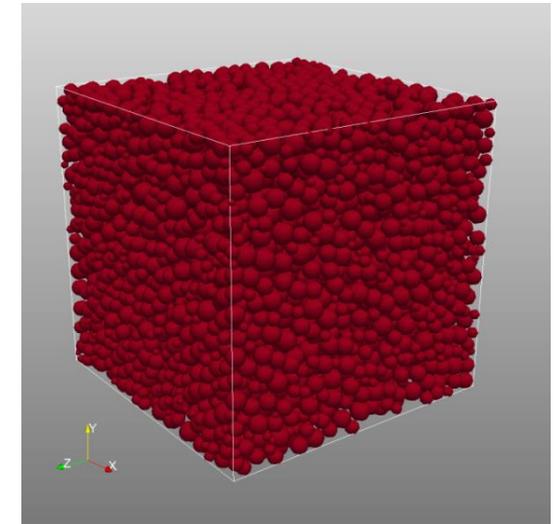
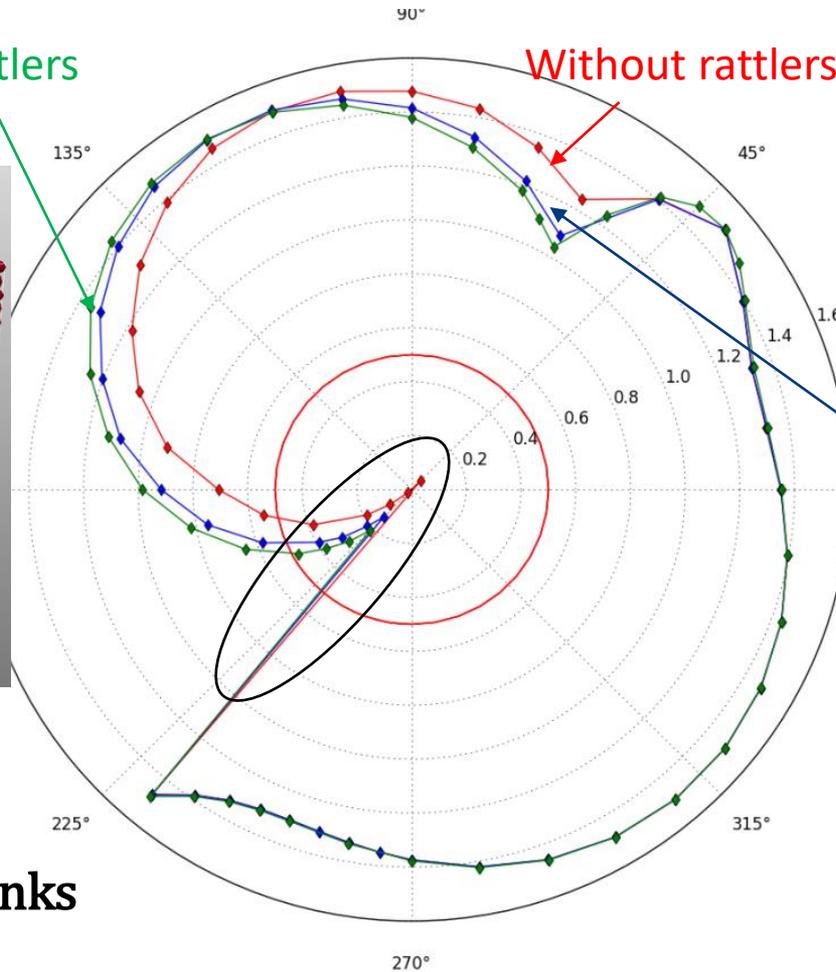
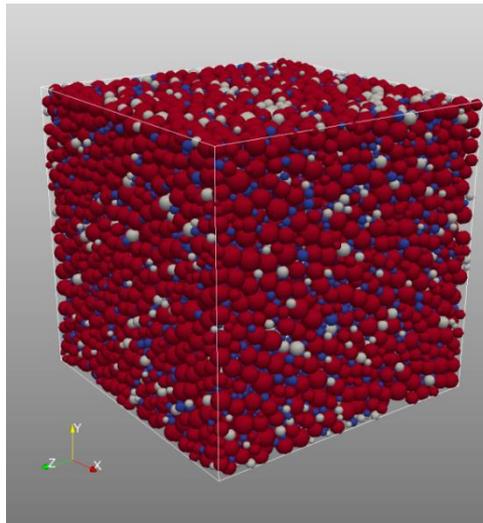
Second order work envelopes

Gudehus (1979)

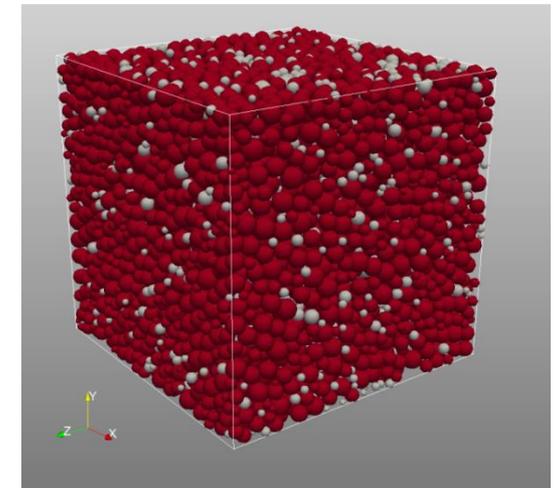
## ➤ Artificial microstructures and stability analysis

With more rattlers

Without rattlers



Original sample



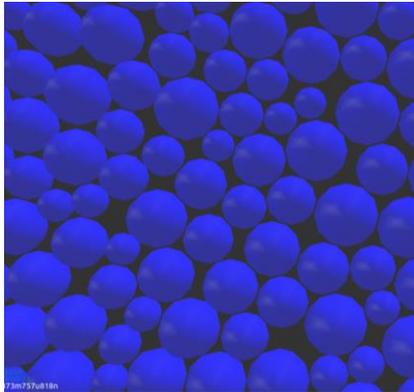
Instability cone shrinks  
with more rattlers



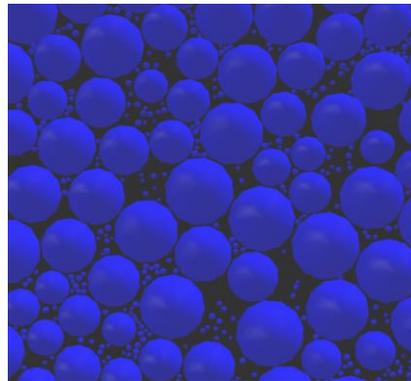
## > Fine content influence

- Gap graded materials
  - $D_{coarse} = 10 D_{fine}$
  - Uniform distribution of coarse and fine radii  $\frac{D_{max}}{D_{min}} = 2$

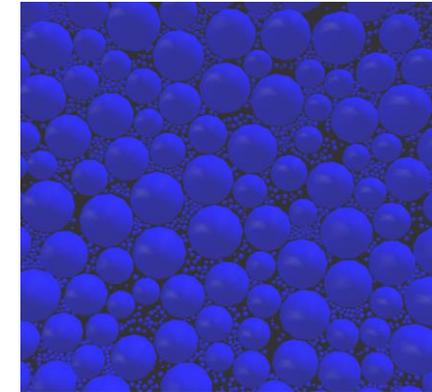
- Three fine contents :



Fine content = 0%



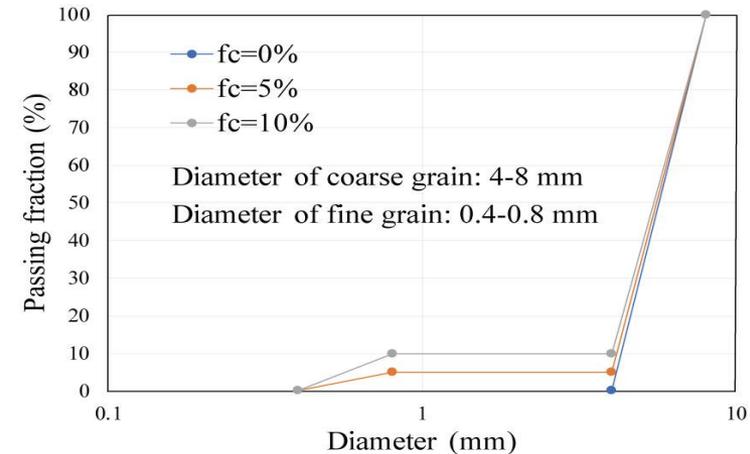
Fine content = 5%



Fine content = 10%

- Same relative density (loose samples) : 10%

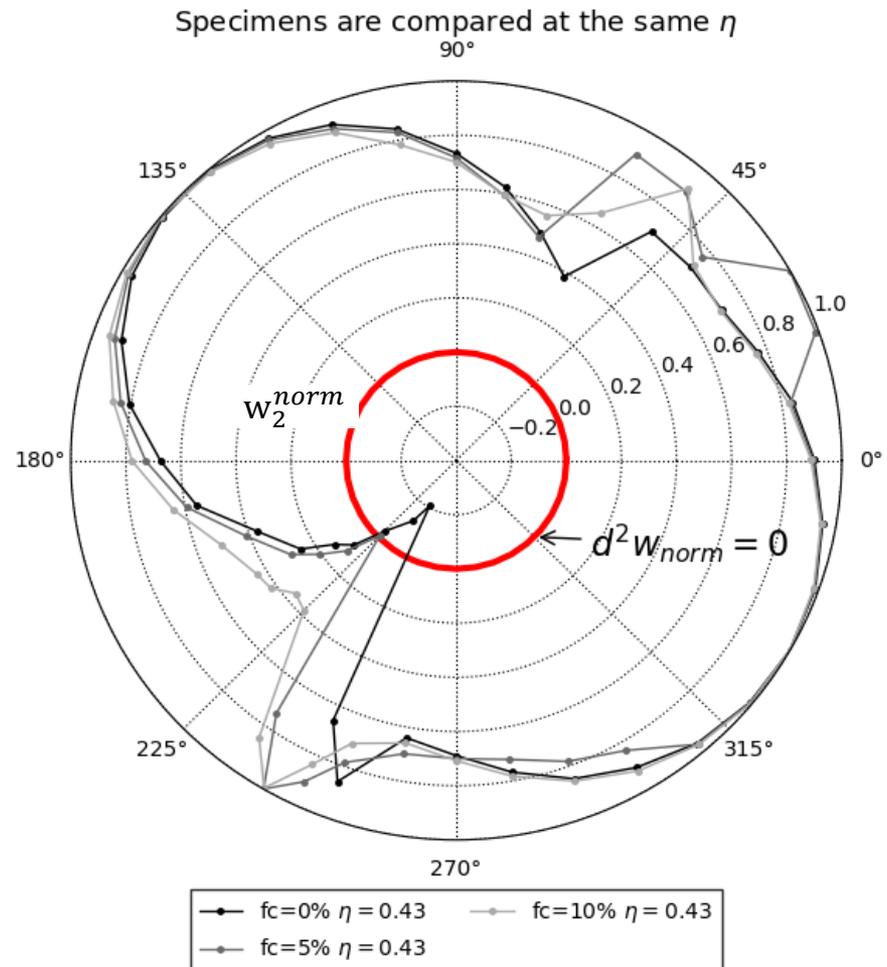
Tao Wang PhD (2020)



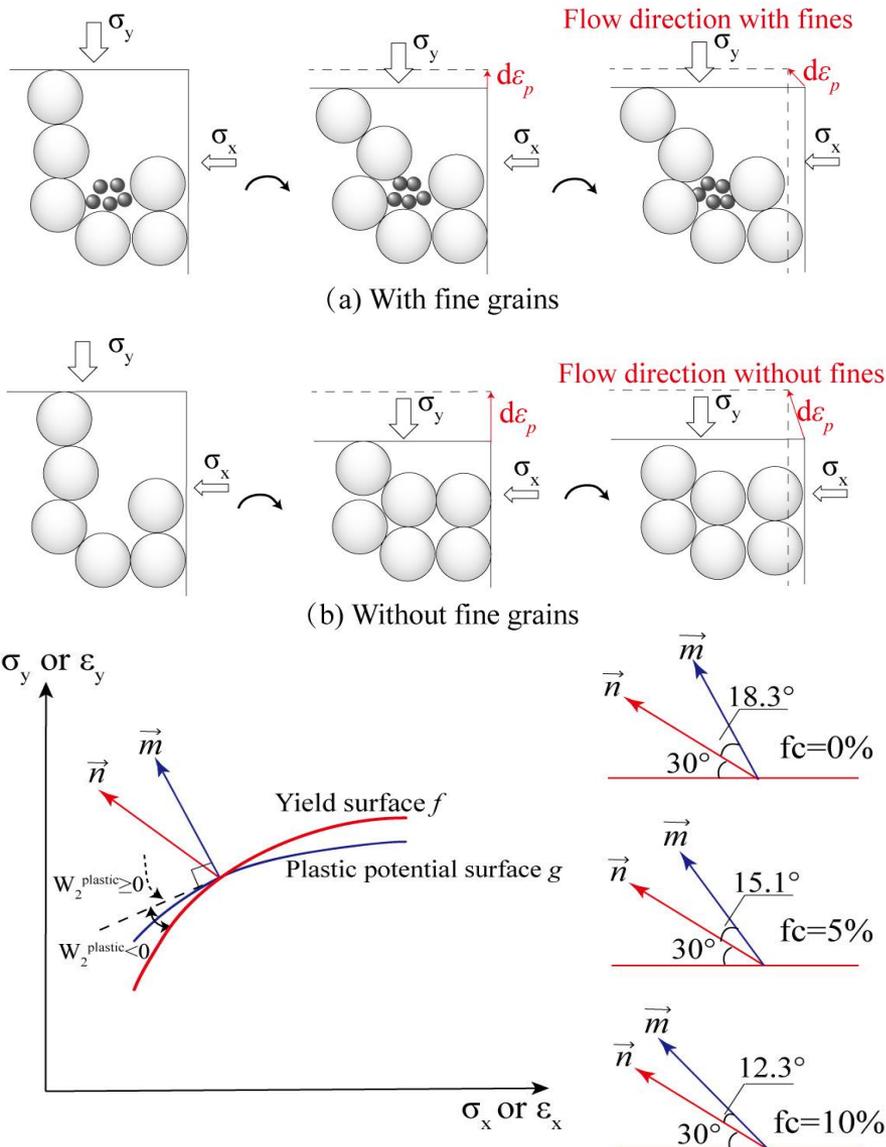
## > Fine stabilization effect

Tao Wang PhD (2020)

1. Cone of instability for  $FC = 0\%$
2. Cone shrinks for  $FC = 5\%$
3. Cone vanishes for  $FC = 10\%$



# ➤ Fine effect on granular plasticity

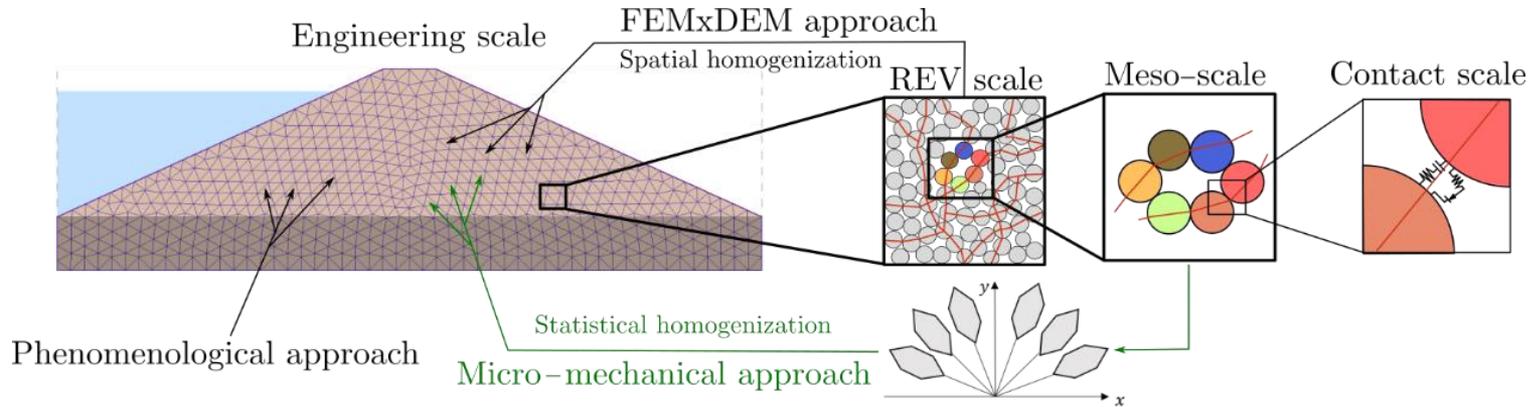


Fine particles have a stabilisation effect:

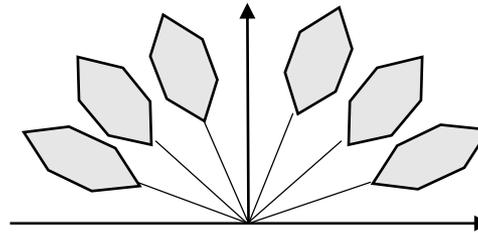
- 1. Increase in dilatancy angle**  
Flow rule direction is modified
- 2. Increase strain hardening**  
Plastic strain is reduced
- 3. No effect on the initial yield surface**  
Friction angle is not modified

NB : Valid for small fine contents  
(underfilled materials)

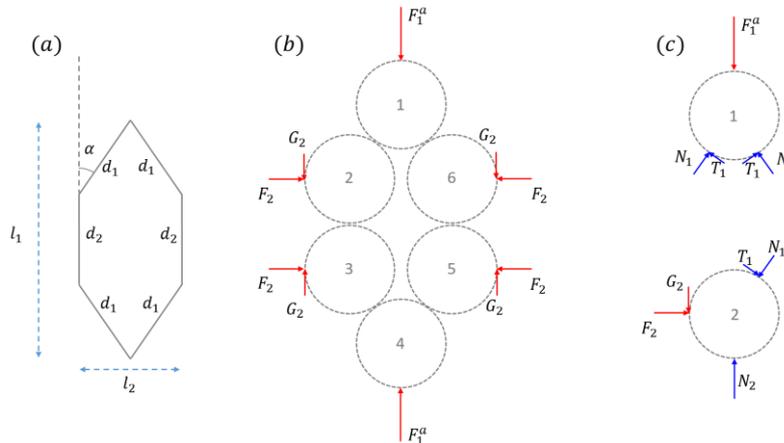
# ➤ Multiscale constitutive behavior: the H-model



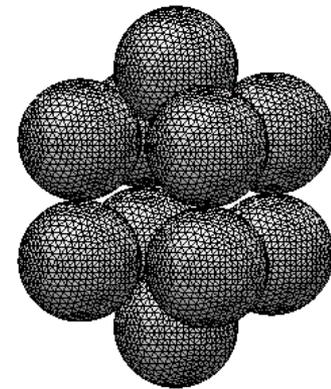
1. Collection of independent mesostructures oriented in different directions



2. Analytical solution of the mechanical equilibrium of each cell

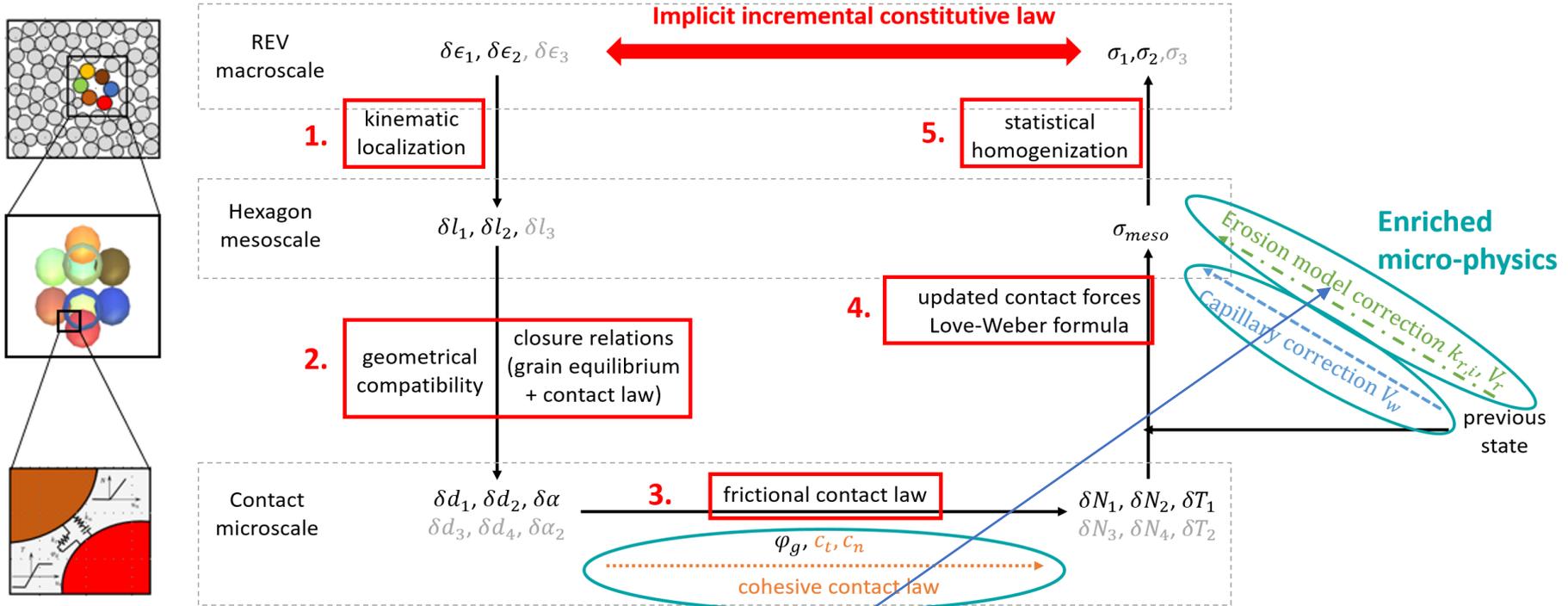


(Nicot and Darve 2011)

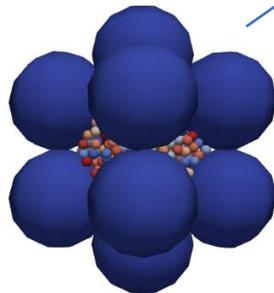


(Xiong et al. 2017)

# ➤ The standard H-model: homogenization scheme



Fine contribution to stress  
 Qirui Ma PhD, (2020)

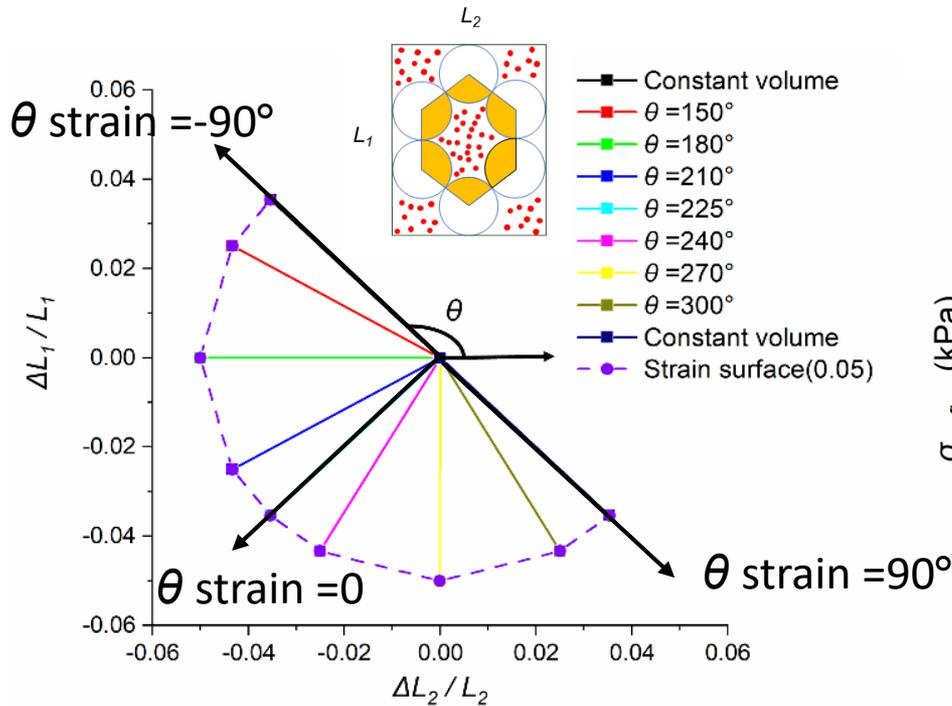


$$\sigma_{ij} = \frac{1}{V} \left( \sum_{c \in c-c} f_i^{c-c} d_j^{c-c} + \sum_{c \in c-f} f_i^{c-f} d_j^{c-f} + \sum_{c \in f-f} f_i^{f-f} d_j^{f-f} \right)$$

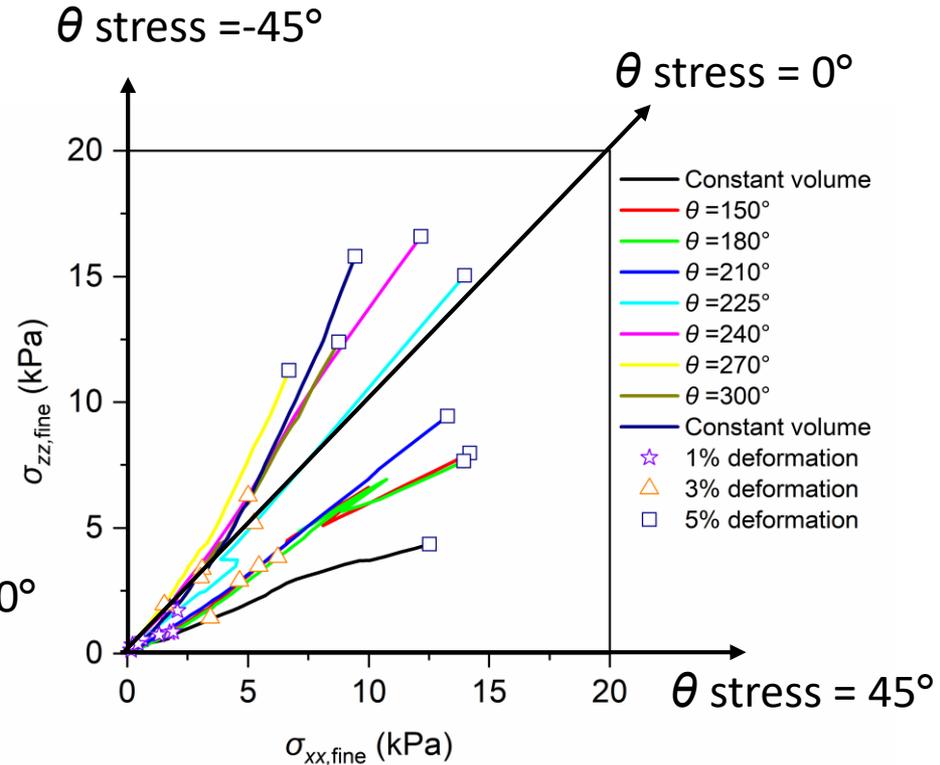
Standard H-model  
 (coarse-coarse contacts)

Fine contribution  
 (coarse-fine and fine-fine contacts)

# Systematic analysis for proportional strain paths

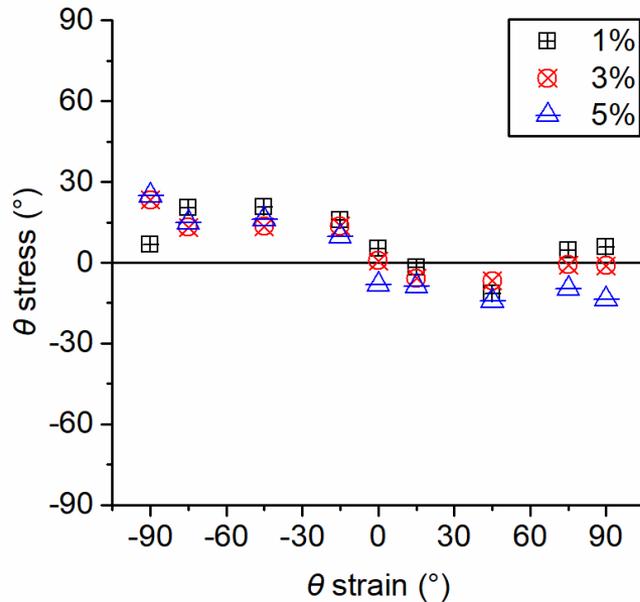


$$(\Delta L_1 / L_1)^2 + (\Delta L_2 / L_2)^2 = 0.05$$



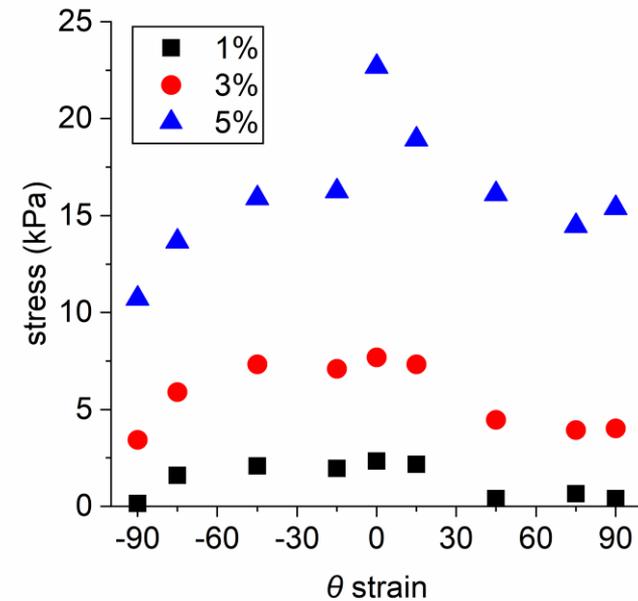
- The stress contribution of fine particles is
- quasi linear with strain intensity
  - Slight anisotropy

## ➤ Proposed model



Equivalent fine stress contribution

$$\sigma = \sqrt{(\sigma_{xx})^2 + (\sigma_{zz})^2}$$



$$d\sigma_f = \left\| d\sigma_f \right\| \begin{pmatrix} \cos\left(\frac{\pi}{4} - \theta_{stress}\right) & 0 \\ 0 & \cos\left(\frac{\pi}{4} + \theta_{stress}\right) \end{pmatrix} \quad \left\| d\sigma_f \right\| = \left\| d\varepsilon \right\| \cos(\omega \theta_{strain})$$

$$\theta_{stress} = \theta_0 \arctan(\beta \theta_{strain})$$

Model parameters depending on fine content and size ratio

## > Conclusion and outlook

### Done

- DEM simulations showed that fine particles have a stabilisation effect:
  1. Increase in dilatancy angle
  2. Increase strain hardening
  3. No effect on the initial yield surface
- An analytical relationship for the stress contribution has been derived from DEM simulations at mesoscale

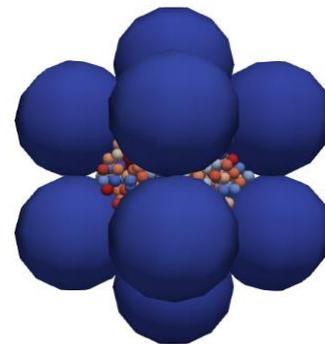
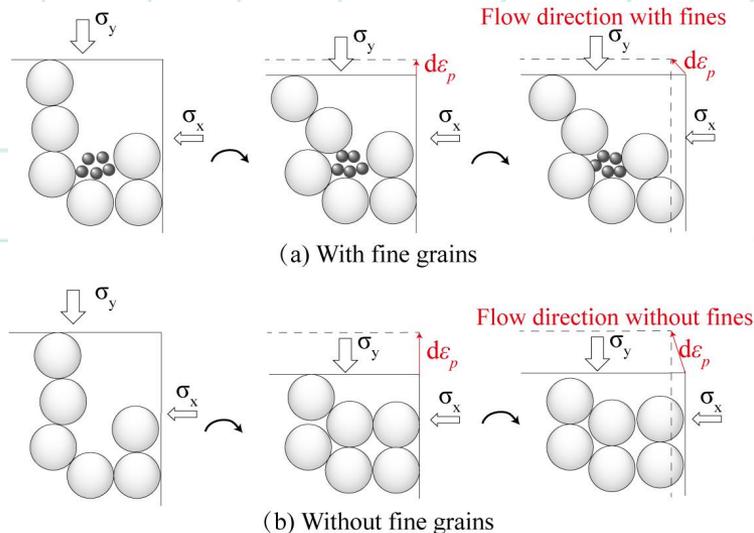
### Remains to be done

- Relate the model parameters to fine content and particle size ratio
- Implement the enriched version of the H-model
- Assess the effect of fluid on fine content (REV and structure scale)
- Assess the impact of suffusion on mechanical stability (material and structure scales)



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